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HIERARCHICAL BAYES SMALL AREA ESTIMATION WITH APPLICATION TO SURVEY DATA: LABOUR MARKET ISSUES FOR GRATUATED STUDENTS FROM THE UNIVERSITY OF TIRANA

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ABSTRACT

Model-based small area estimation methods are so encountered in practice because of the increasing demand for effective and precise estimates for small regions or areas. Usually sample surveys are designed to provide reliable estimates for large regions or areas and the direct survey estimates provide reliable estimates of the parameter of interest for those large areas.
In this paper, we study small area estimation using area-level models such as Fay-Herriot (Fay and Herriot, 1979) when sampling variances are known and You-Chapman (You and Chapman, 2006) when variance modeling is needed. We consider hierarchical Bayes afterwards that extends the two previous models. The proposed models on the survey data, for the estimation of the employment rate of the graduated students in the last four years in the University of Tirana, are implemented using Gibbs sampling method for fully Bayesian inference.
It is used software R for all the computational results, especially 'BayesSAE' package that it is related directly with Bayesian hierarchical models under different linking models.

Keywords: area-level models, linking models, hierarchical Bayes, **DIC**.

INTRODUCTION

Recently the model-based estimations are widely used in practice to provide indirectly efficient estimates for small area. In general, models for small area are classified into two groups: unit level models and area level models. Unit level models are usually based on observation units from surveys and the auxiliary variables of each survey, while area level models are based on direct estimators of aggregated surveys from both unit level data and the auxiliary variables at the level areas. In this way, typically, area level models have the ability to protect the confidentiality of microdata. The other advantage of the area level models is that they take into consideration the construction of the survey by using direct survey estimators and variance estimators related to the design-based variance estimates. Many area level models are proposed to increase the accuracy of the estimators taken directly from surveys. An important model among them is the Fay-Herriot model (Fay and Herriot, 1979) that is a model at baseline levels. The Fay-Herriot model is a sample model for the direct survey estimators and a linking model with the small area parameters that interests us.
For the Fay-Herriot model, the sample variance is usually assumed to be known while in practice, smooth Fay-Herriot model variance estimators are used and after that are considered as known. The smoothing of the sample variance estimators is usually done by using the generalized variance function (Dick, 1955). Most recently, Singh, Folsom and Vaish (2005) suggest the use of generalized design effects in the covariance matrix procedure. You (2008) used the common design effects model to smooth the sample variance. The assumption of the linear linking model and the known values of the sample variances are the two major constraints of the Fay-Herriot model. For this reason, many advanced models have been proposed for different applications in practice. Another limitation is that it is only a cross sectional model. In many applications, temporal and spatial correlations can be used to improve the effectiveness of model-base estimators on direct survey estimators.
Once complex models are provided for the proposed small areas in applications, the Bayesian hierarchical method using generated samples from Gibs algorithm is widely used to overcome the calculation difficulties in obtaining the posterior estimators of the small area parameters. An advantage of the Bayesian hierarchical method is that it relatively straight forward and the inferential analysis of small area parameters is more efficient unlike EBLUP (Empirical Best Linear Unbiased Predictor), even when the sample sizes for specific areas are very small which is the typical problem for the small area estimators.

METHODOLOGY

Small area estimators seek to develop the accuracy of the estimators when the standard methods are not very efficient. Thus, the small area estimator method produces estimators for areas that do not have a suitable and efficient estimator. The concept of this theory can be a bit confusing it does not require areas to be small but it is small the number of the statistical units. Usually small areas are called small spaces because they refer mainly to geographical spaces or different demographic or socio-economic groups.

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The objective of this paper is to present the linking models that relate the parameter of interests to a regression model with specific area random effects using Fay-Herriot model, You-Chapman model (You and Chapman, 2006) and spatial model.

1. Fay-Herriot model

The Fay-Herriot model is divided into two components that are the sampling model for direct estimation and the linking model for the parameters we are interested in. The first component includes direct estimation that comes from survey and the corresponding sampling variance. It is then supposed that the variance is known and used afterwards in the model. The second component relates the parameters with random effects, that are usually considered as identically normal distributed and independent.
If θ_i is the unknown parameter of interest for the i-th area, the Fay-Herriot model assumes that is related to area specific auxiliary data $x_i = (x_{i1}, x_{i2}, ..., x_{ip})$ to a linear regression model:

$$\theta_i = x_i^{'}\beta + v_i, \; i=1, 2,..., m, \quad (1)$$

where m is the total number of areas, $\beta = (\beta_1, \beta_2, ..., \beta_p)'$ is $px1$ vector of the regression coefficient and v_i are specific area random effects that are identically distributed with $E(v_i)=0$ and $var(v_i)=\sigma^2_v$. This is a linking model for the parameter θ_i. The Fay-Herriot model may assume that v_i are normally distributed and the direct survey estimator y_i is usually design-unbiased for θ_i and it can be defined as in (2):

$$y_i = \theta_i + e_i, \; i=1, 2,..., m, \quad (2)$$

where e_i-s are the sampling errors related with the direct estimator y_i, that are assumed to be independent variables with normal distribution with mean $E(e_i \mid \theta_i) = 0$ and the sampling variance $D(e_i \mid \theta_i) = \sigma_i^2$. The model (2) is the direct survey estimator y_i. From the combination of the information of (1) and (2), it is taken the model (3):

$$y_i = x_i^{'}\beta + v_i + e_i, \; i=1, 2,..., m. \quad (3)$$

In the Fay-Herriot model (3) it is usually made a strong assumption that the sampling variances are known, but instead it is used direct sampling variance estimates. As the direct sampling variance estimates can be instable in small sample sizes then a generalized variance function is used for the sampling variance (ex. Dick, 1995). In the last years it has been developed and used a method of smoothing design effects for smoothed variance estimators (Singh, Folsom and Vaish in 2005, You in 2008; Liu, Lahiri and Kalton in 2007). For example You, applied an equal design effects model to obtain smooth estimates of the sampling variances and the design effect for the i- th area is the formula (4):

$$deff_i = \frac{s_i^2}{s_{ri}^2}, \; i=1, 2,..., m, \quad (4)$$

where s_i^2 is the unbiased direct estimate of the sampling variance based on the complex sampling design and s_{ri}^2 is the sampling variance estimate under the assumption of a simple random sampling design. In this way a smoothed factor deff can be obtained by $deff = \frac{\sum_{i=1}^{m} deff_i}{m}$ and the smoothed sampling variance can be $\tilde{\sigma}_i^2 = s_{ri}^2 * deff$.

If the smoothed estimates of the sampling variances are not used then the sampling variance can be modeled directly. Based on Wang and Fuller (2003) and You and Chapman (2006), it is assumed σ_i^2 unknown and it is estimated by the direct unbiased estimator s_i^2 that is independent with the direct survey estimator y_i. They proposed other assumptions such as $d_i s_i^2 \sim \sigma_i^2 \chi_{d_i}$ where $d_i = n_i - 1$ and n_i is the sample size of area i. The full hierarchical Bayes approach with Gibbs sampling method was used by You and Chapman in 2006, which takes into consideration the extra uncertainty associated with the estimation of σ_i^2.

2. Spatial model

A simple and obvious way to integrate spatially correlated random effects in the linking model is by adding the random effect u_i in the linking model (1) as in (5):

$$\theta_i = x_i^{'}\beta + v_i + u_i, \quad (5)$$

where u_i follow the intrinsic conditional autoregressive model:

$$u_i \mid u_{-i} \sim N\left(\frac{\sum_{j\neq i} w_{ij}u_j}{\sum_{j\neq i} w_{ij}}, \frac{\sigma_u^2}{\sum_{j\neq i} w_{ij}}\right) \quad (6)$$

Where u_{-i} is the value of spatial random effects u_j in all the other areas with $j\neq i$, the weights w_{ij} are fixed constants and σ_u^2 is an unknown variance component. A practical choice is $w_{ij}=0$ unless i and j are neighboring areas, in which $w_{ij}=1$. The model (5) was proposed by Besag, York and Mollie in 1991 in order to separate spatial effects the overall heterogeneity in the areas.

Usually it is difficult in practice to choose between the unstructured model as model (1) and a spatially structured model as model (6). In the model (5), the posterior inference about the spatial is based on the proportion of the total variation in the sum of v_i+u_i for each component. The corresponding joint distribution is improper (undefined mean and infinite variance) even though the univariate conditional distributions of the spatial component are well defined. A risk problem for model (5) is when only the sum of the random effects v_i+u_i is well defined (Best et al, 2005).

An alternative spatial parameterization may be considered, as it was proposed by Leroux, Lei and Breslow in 1999 and later from MacNab (2003), to avoid identifiability problems of model (5). If we consider the model: $\theta_i = x_i'\beta + b_i$ where $b = (b_1, b_2, ..., b_m)'$, according to Leroux et al. (1999) and MacNab (2003), we set the following conditional autoregressive (CAR) model on the area with spatial effects ***b***:

$$b \sim MVN(0, \sum(\sigma_b^2, \lambda)) \qquad (7)$$

$$\sum(\sigma_b^2, \lambda) = \sigma_b^2 D^{-1},\ D = \lambda R + (1-\lambda)I \quad (8)$$

where σ_b^2 is a spatial dispersion of the parameter λ which is a spatial autocorrelation parameter and $1 \le \lambda \le 1$, I is an identity matrix of dimension *m*, *R* is the commonly known the neighborhood matrix that has *i-th* diagonal element equal to the number of the neighbors of area *i* and the other elements equal to *-1* if the areas are neighbors and *0* otherwise. The CAR model (7) and (8) can be expressed:

$$b_i \mid b_{-i} \sim N\left(\frac{\lambda}{1-\lambda+\lambda\sum_{j\neq i} w_{ij}}\sum_{j\neq i} w_{ij} v_j, \frac{\sigma_b^2}{1-\lambda+\lambda\sum_{j\neq i} w_{ij}}\right) \quad (9)$$

If $\lambda=1$ then the autoregressive model is the intrinsic model (6) and if $\lambda=0$ the CAR model becomes the independent linking model (1) that assumes independence on area-specific random effects v_j. The conditional mean and variance of $b_i|b_{-i}$ are the weighted sum of the overall moment from the linking model (1) and local smoothing moments from the instrict autoregressive model:

$$E(b_i \mid b_{-i}) = \frac{1-\lambda}{1-\lambda+\lambda\sum_{j\neq i} w_{ij}} \times 0 + \frac{\lambda\sum_{j\neq i} w_{ij}}{1-\lambda+\lambda\sum_{j\neq i} w_{ij}} \frac{\sum_{j\neq i} w_{ij} b_j}{\sum_{j\neq i} w_{ij}}$$

$$\mathrm{var}(b_i \mid b_{-i}) = \frac{1-\lambda}{1-\lambda+\lambda\sum_{j\neq i} w_{ij}} \times \sigma_b^2 + \frac{\lambda\sum_{j\neq i} w_{ij}}{1-\lambda+\lambda\sum_{j\neq i} w_{ij}} \frac{\sigma_b^2}{\sum_{j\neq i} w_{ij}}$$

This shows that the model (7), (8) is a balance between the linking model (1) and the CAR model (6). The spatial correlation parameter λ measures the spatial effects for local smoothing of the neighborhood areas.

3. Hierarchical Bayes and inferential analysis

We will estimate the parameter θ_i applying a Hierarchical Bayes (HB), which totally differs from EBLUP and Empirical Bayes. Moreover, the HB approach can deal with complex small

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area models using Monte Carlo Markov Chain (MCMC) method for simulations in order to overcome the computational difficulties of multi dimensional posterior quantitaties.

Let we denote $y=(y_1, y_2, ..., y_m)'$, $\theta=(\theta_1, \theta_2, ..., \theta_m)'$ and $X=(x_1, x_2, ..., x_m)'$. We will propose three hierarchical models: the basic Fay-Herriot model under the assumption that σ_i^2 are assumed known, the You-Chapman model where σ_i^2 is unknown and the extension You-Chapman model with spatial random effects with unknown sampling variances.

Model1: Fay and Herriot model (Fay and Herriot 1979, Rao 2003) can be expressed:

- $y_i \mid \theta_i \sim N(\theta_i, \sigma_i^2 = \tilde{\sigma}_i^2)$, i=1, 2, …, m;
- $\theta_i \mid \beta, \sigma_v^2 \sim N(x_i'\beta, \sigma_v^2)$, i=1, 2, …, m;
- The priors for the parameters (β, σ_v^2) are: $\pi(\beta) \propto 1$, $\pi(\sigma_v^2) \sim InversG(a_0, b_0)$ where a_0 and b_0 are chosen to be very small and considered as known in order to reflect less in σ_v^2.

Model2: You-Chapman model (You and Chapman, 2006) can be expressed:

- $y_i \mid \theta_i, \sigma_i^2 \sim N(\theta_i, \sigma_i^2)$, *i=1, 2, …, m;*
- $d_i s_i^2 \mid \sigma_i^2 \sim \sigma_i^2 \chi_{d_i}^2$, $d_i = n_i - 1$, *i=1, 2, …, m;*
- $\theta_i \mid \beta, \sigma_v^2 \sim N(x_i'\beta, \sigma_v^2)$, *i=1, 2, …, m;*
- The priors for the parameters $(\beta, \sigma_v^2, \sigma_i^2, i=1,2,...m)$ are: $\pi(\beta) \propto 1$, $\pi(\sigma_v^2) \sim InversG(a_0, b_0)$, $\pi(\sigma_i^2) \sim InversG(a_i, b_i)$ for *i=1, 2, …, m*, where a_i and b_i $0 \le i \le m$ are chosen to be very small and known in order to reflect less in σ_v^2 and σ_i^2.

Model3: An extension of You-Chapman model with unknown sampling variances and with area level CAR, can be expressed:

- $y \mid \theta, \sigma_1^2, \sigma_2^2, ..., \sigma_m^2 \sim MVN(\theta, E)$, where E is a matrix with diagonal elements σ_i^2.
- $d_i s_i^2 \mid \sigma_i^2 \sim \sigma_i^2 \chi_{d_i}^2$, $d_i = n_i - 1$, *i=1, 2, …, m;*
- $\theta \mid \beta, \sigma_v^2 \sim MVN(X\beta, \sigma_v^2 D^{-1})$, $D = \lambda R + (1-\lambda) I$;
- The priors for the parameters $(\beta, \lambda, \sigma_v^2, \sigma_i^2, i=1,2,...m)$ are: $\pi(\beta) \propto 1$, $\pi(\lambda) \sim U(0,1)$, $\pi(\sigma_v^2) \sim InversG(a_0, b_0)$, $\pi(\sigma_i^2) \sim InversG(a_i, b_i)$ for *i=1, 2, …, m*, where a_i and b_i $0 \le i \le m$ are chosen to be very small and known constants.

RESULTS

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1. **Data description**

The periodic study of the dynamics of the labor market is now a necessity for determining the fairest of the development strategy of study programs in all the study cycles. The need for measuring the frequent performance is related to the dynamics of the structural changes that experiences the labor market in Albania as a reflection of changing the structure of economy, its modernization, the growth of international economic links, the development of social relations etc. For this reasons it was taken a survey to study the labor market issues for graduated students from Master degrees in all the faculties of the University of Tirana at the last four years, from 2014 to 2017. It aims to analyze the data for youth employment who graduate from the University of Tirana and it provides a panorama of the characteristics the university offers for the labor market.

Table 1. The results of survey divided by Faculty, graduating year and employment

Faculty	**Graduating Year**				**Actual employment**		**Total cases**
	2014	2015	2016	2017	Yes	No	
Faculty of Justice	11	10	27	46	58	36	94
Faculty of Economy	63	128	117	236	479	65	544
Faculty of History and Philology	8	14	18	26	37	29	66
Faculty of Foreign Languages	19	18	28	35	80	20	100
Faculty of Natural Sciences	27	17	21	80	98	47	145
Faculty of Social Sciences	6	7	61	108	99	83	182

In Table 1. is given a description of the collected data from the survey and the main topics we are interested in such as the description of the Faculties of the University of Albania, the graduating year of the participants in the survey and the actual position of employment.

2. **'BayesSAE' package and the implementation**

We are going to use 'BayesSAE' package in R to make several analysis for specific small area-level models for the bayesian approach. The package provides a variety of methods from Rao (Rao, 2003) and other research articles to deal with several specific small area area-level models in the Bayesian analysis. The function BayesSAE specifies the model and we obtain MCMC posterior draws of specific small-area level models from different types of priors and defined by the sampling model and the linking model. Models provided range from the basic Fay-Herriot model to its improvement such as You-Chapman models, unmatched models ore spatial models. There is also integrated a model checking criteria to find the best model or if it is suitable or not.
Let we denote θ_i the true employment rate for the i-th area that is one of the Faculties of the university of Tirana and we also include four auxiliary variables used in the model that are:

the graduating year, the experience with training issues (question 10 of the survey questionnaire), the sample size for each area and the direct variance estimator.
The first hierarchical Bayesian analysis is based on the Fay- Herriot model and the function used is BayesSAE in R after 5000 simulations with MCMC, as follows:

```
Result1 <- BayesSAE(meanp~Graduation+Q10+Ni|varp, data = DATA, mcmc = 5000)
```

The results of the regression coefficients β_1, $\beta_{2,}$ $\beta_{3,}$ β_4 of the linking problem under the Fay-Herriot model are shown in Table 2.

Table 2. The results of basic Fay-Herriot model (FH) of the linking model

Coefficient	Coefficient Value	Coefficient mean	Coefficient SD
Beta1	39.7696244	3.977e+01	1.921e+01
Beta2	-0.0194523	-1.945e-02	9.523e-03
Beta3	0.0036264	3.626e-03	2.167e-03
Beta4	0.0005822	5.822e-04	3.448e-05

The second model of HB is based on the You-Chapman Model and it can be obtained as follows:

```
Result2 <- BayesSAE(meanp~Graduation+Q10+Ni |varp, data = DATA, mcmc = 
5000, innov = "t",+ df = rep(1130, m))
```

The results of the regression coefficient $\beta_1, \beta_{2,} \beta_{3,} \beta_4$ when You-Chapman model id used, are shown in table 3 including their mean and standard deviation:

Table 3. The results of basic You-Chapman(YC) model of the LM

Coefficient	Coefficient Value	Coefficient mean	Coefficient SD
Beta1	34.0040028	1.264e+01	1.076e+01
Beta2	-0.0165718	6.266e-03	-2.925e-02
Beta3	0.0012676	3.443e-03	-6.316e-03
Beta4	0.0004995	3.875e-05	4.463e-04

The third model of HB is based on the You-Chapman model with spatial effects is taken from the command in R:

```
Result3=BayesSAE(meanp~Graduation+Q10+Ni  |varp,  spatial  =  TRUE,  prox  = 
prox, mcmc = 5000, data = DATA)
```

The coefficient of the regression of the linking models differ from the other two models, they are shown in table 4:

Table 4. The results of You-Chapman with spatial effects model of LM

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Coefficient	Coefficient Value	Coefficient mean	Coefficient SD
Beta1	-7.9768788	-7.977e+00	1.060e+01
Beta2	0.0042400	4.240e-03	5.252e-03
Beta3	0.0042087	4.209e-03	2.866e-03
Beta4	0.0005306	5.306e-04	4.073e-05

The means and the standard deviations of variance residuals for each implemented models are shown in Table5:

Table 5. The results of variance residuals for each model

Model	Mean	SD
Fay-Herriot	0.0011317	0.0012011
You-Chapman	0.0007740	0.0008218
You-Chapman with spatial random effects	0.0013429	0.0013805

The final estimates for each area (each of the six faculties of university of Tirana) including the computational results for the three models are shown in Table 6.

Table 6. Theta values for each model

THETA	Sample means	Fay-Herriot	You-Chapman	You-Chapman with spatial random effects
Θ_1	0.620	0.607	0,632	0,641
Θ_2	0.880	0.885	0,880	0,871
Θ_3	0.560	0.593	0,619	0,624
Θ_4	0.800	0.617	0,641	0,637
Θ_5	0.680	0.639	0,660	0,669
Θ_6	0.540	0.660	0,679	0,680

The first column has the sample means and the other columns the estimates value taken after 5000 simulations with MCMC and the different linking models that were taken into the study. All the theta estimates are taken under Rao-Blackwellization of θ under the 5000 simulation taken from MCMC. As it can be seen the Faculty of Economy estimators differs less than the others from the sample mean, the estimators from the other faculties differs more.

3. Hierarchical Bayes best model

The Deviance Information Criterion (DIC) is a hierarchical modeling generalization of the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC). It was proposed by Spigelhalter, Best, Carlin and van der Linde (2002) and it is very useful to compare mixed effects Bayesian models when the posterior distributions have been obtained

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by Marcov Chain Monte Carlo (MCMC) simulations. It is based on the deviance of the model *D(θ)*, which is equal to minus twice the log-likelihood of the model and DIC is calculated:

$$DIC = D(\hat{\theta}) + 2p_D,$$

where $D(\hat{\theta})$ is the estimated deviance of the model from the posterior mean of the model parameters that summarizes the goodness of fit of the model, p_D is the effective number of parameters that it is related with complexity of the model and it is calculated $p_D = \overline{D}(\theta) - D(\hat{\theta})$ where $\overline{D}(\theta)$ is the posterior mean of the deviance of the model. So the DIC is defined as the summarization of the goodness of fit of the model and the complexity of the model. The best models have low value of DIC. The computation is made directly for $D(\hat{\theta})$ and p_D after the running of Gibbs sampling by taking the sample mean of the simulated values of $D(\theta)$ minus the plug-in estimate of the deviance $D(\hat{\theta})$.

The results of the three models with the DIC values for each of them are shown in Table 7 and the differences between them are really consistent.

Table 7. The comparison of DIC values for each Hierarchical model

Model	**DIC value**
Fay-Herriot	24.03231
You-Chapman	14.93189
You-Chapman with spatial random effects	12.03293

The model 3 with spatial random effects is obviously better than the two other proposed models.

DISCUSSION

Referring to Brown, Chambers and Heasman (2001), it can be considered a simple method of regression analysis for the direct estimators and the hierarchical Bayes model-based estimators for any evaluation of possible bias of the model based estimators under the proposed model and the direct survey estimators. If the model-based estimates are close to the true values of the small area then the direct survey estimates are considered random variables with expected values equal to the model-based estimates values. Even You (2008) used the regression analysis for the diagnostic of model bias.

CONCLUSIONS

In this paper we have discussed three different area level models, the well-known Fay-Herriot model when the sampling variance is assumed to be known, the You-Chapman model in which the sampling is unknown where it is substituted by direct estimators and the You-Chapman model with spatial random effects where it is supposed a small area spatial correlation between the different areas. For both Fay-Herriot and You-Chapman model, the area random effects are assumed to be *iid* normal random variables in order to explain better the area heterogeneity effects. Instead, in the spatial random effect model it is included the small area sampling models and a spatial correlation linking model that includes both unstructured heterogeneity between areas and the spatial random effects of the neighboring areas, so there is no need to specify the spatial correlation parameter in the model because it will be estimated from the data.

In the data analysis we compared the different models to estimate the rates of employment between graduated student from different faculties of the University of Tirana and the results of Bayesian model comparison show that the model with spatial random effects is the best among the classic Fay-Herriot and You-Chapman model.

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